Loss of Crack-Tip Constraints for Shallow Cracks in Bending

J.F. Zarzour, D.-W. Yuan, and M.J. Kleinosky

Single-edge notched bars (SENB) in bending with *a/W* ratios ranging from 0.05 to 0.5 are examined for fracture toughness in terms of the *J*-integral approach. The corresponding results indicated that, for a *a/W* ratios less than 0.3, there is a significant loss of *J*-dominance, which is attributed to the impact of plastic deformation on the cracked face. For larger *a/W* ratios, *J*-dominance is maintained into a large-scale yielding regime. According to the recently developed two-parameter criterion (*J,Q*), compressive *Q*-stress was interpreted as an indication of low crack-tip stress triaxiality for shallow cracks, whereas positive *Q*-stress was associated with high crack-tip stress triaxiality for deep cracks. For the material properties and specimen geometries considered herein, a fracture toughness locus is constructed in terms of the (*J-Q*) parameters for each of the *a/W* ratios. The overall fracture data, which are in agreement with those predicted by other approaches, appear to provide a rigorous framework for interpreting the effect of loss of crack-tip constraint in elastic-plastic fracture analysis.

Keywords
fracture toughness, low constraint, shallow cracks

1. Introduction

UNDER small-scale yielding conditions, the asymptotic stress and strain fields ahead of a crack can be characterized by the HRR fields after Hutchinson[1,2] and Rice and Rosengren.[3] These fields that are valid within the plastic zone at distances of the order of the crack tip opening displacement (CTOD) from the crack tip take the form:

$$\sigma_{ij} = \left[\frac{J}{\varepsilon_0\, \sigma_0\, \alpha\, In\, r}\right]^{1/(n+1)} \tilde{\sigma}_{ij}(n,\theta)$$

$$\varepsilon_{ij} = \frac{\alpha\, \sigma_0}{E}\left[\frac{EJ}{\alpha\, \sigma_0^2\, In\, r}\right]^{n/(n+1)} \tilde{\varepsilon}_{ij}(n,\theta) \qquad [1]$$

Where In is an integration constant that depends on the hardening exponent, n; $\tilde{\sigma}_{ij}$ and $\tilde{\varepsilon}_{ij}$ are dimensionless functions of n and θ; and σ_0 is the yield stress in uniaxial tension.

According to Eq 1 for a given material property, the *J*-integral developed by Rice[4] determines the intensity of the HRR fields. However, it must be recognized that the formulation of the HRR fields is based on a small strain theory (up to 10%) and, therefore, does not recognize the effect of the blunted crack tip on the stress fields. The finite-element results of McMeeking and Parks[5] have shown that the crack tip blunting is accompanied by local stress unloading at distances on the order of twice the CTOD (Fig. 1). Thus, within this sector where stresses are influenced by large strains, the HRR fields are not valid. Although the solution of the HRR fields is not applicable in this case, fracture criteria that are uniquely characterized by a single parameter such as the critical value of *J*-integral (*Jc*), stress-intensity factor (*Kc*), or the crack-tip opening displacement (δc) are valid as long as the HRR fields given by Eq 1 are satisfied within any forward sector ahead of the crack tip. In this case, (*Jc*), (*Kc*), or (δc) are size-independent measures of fracture toughness.

Under full plasticity, the limitation of *J*-dominance was analyzed by McMeeking and Parks[5] and Shih and German,[6] who used finite-element analysis to compare the small-scale yielding solution to the local stress fields near the blunted crack tip. It was concluded that, for deeply cracked geometries in bending, the local stress fields scale similarly to the small-scale yielding solution as long as the uncracked ligament is greater than 25 (J/σ_0). In other words, *J*-dominance is maintained for (J/σ_0) ratios larger than 25.

A particularly simple treatment for characterizing the observed toughness differences between deep and shallow flaw geometries has been recently introduced by O'Dowd and Shih[7] and Betegon and Hancock.[8] Presently (Part I), three-point bending bars are analyzed following the two-parameter

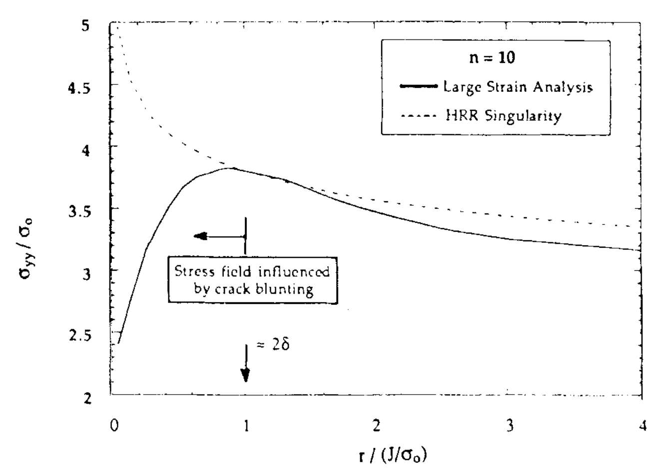

Fig. 1 Stress unloading due to the crack-tip blunting effect. After McMeeking and Parks.[5]

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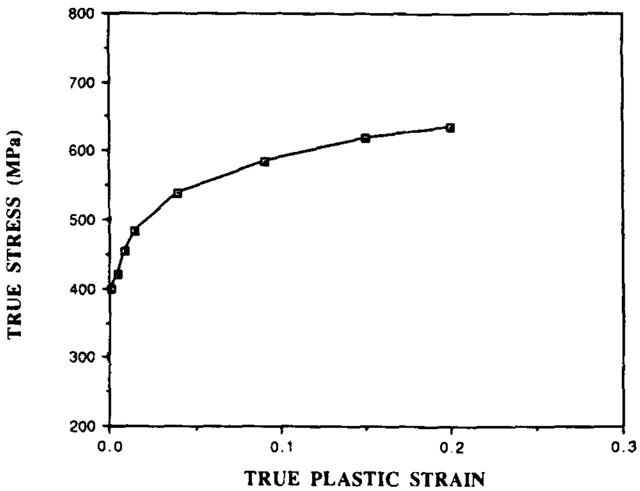

Fig. 2 Tensile material properties.

criterion (J-Q) of O'Dowd and Shih.[7] Future work (Part II) will include the case where undermatched welds are located in the central region of the three-point bend bars.

The main focus of the present study is to highlight the shallow crack effect on the effective fracture toughness, characterized by a critical combination of (Jc,Qc). This implementation of the (J,Q) methodology as indicated by O'Dowd and Shih[7] necessitates the construction of toughness loci for cracks having various aspect ratios (a/W = 0.05 to 0.5) as will be shown in the following sections.

2. Material Properties

The uniaxial stress-strain behavior for the material with constants ($\alpha = 1$, $n = 10$) was obtained from the standard Ramberg-Osgood power law hardening relationship:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \quad [2]$$

The linear elastic portion of the true-stress/true-strain curve is characterized by a yield strain of magnitude $\varepsilon_0 = \sigma_0/E$ = 0.0019. Young's modulus (E) is equal to 208,000 MPa (30.2 × 10^6 psi), and the uniaxial yield stress in tension (σ_0) is equal to 400 MPa (58 × 10^3 psi). The corresponding uniaxial true-stress/true-plastic curve is shown in Fig. 2.

3. Finite-Element Model

The specimens considered in this work are single-edge notched bending (SENB) (standard ASTM E813 fracture toughness geometry). Several crack aspect ratios (a/W) were considered so that a comparison between shallow and deep cracks could be established. Details of the specimen geometries are given in Fig. 3.

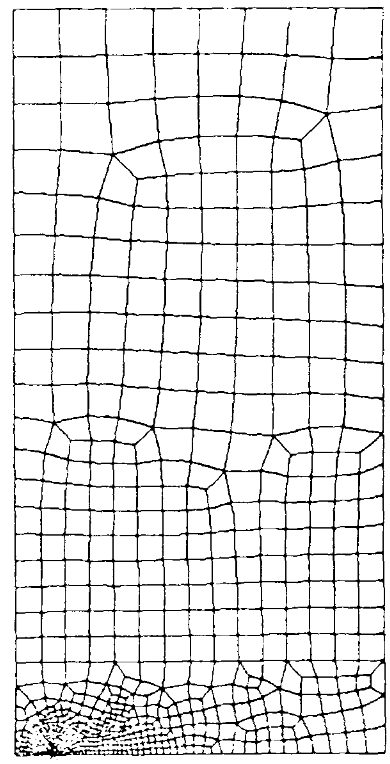

Fig. 3a Typical two-dimensional finite-element mesh for SENB specimen (half model).

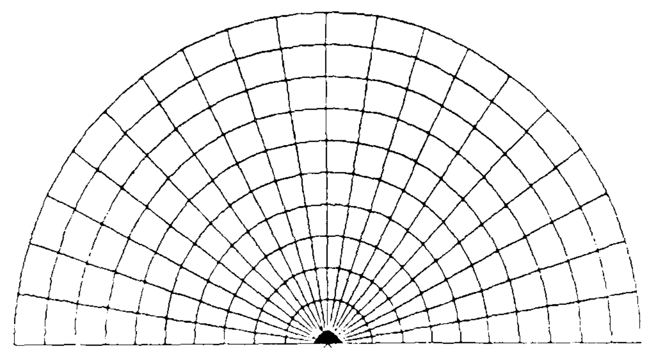

Fig. 3b Focused mesh around the crack tip.

The finite-strain, elastic-plastic analyses of the SENB specimens were performed using the finite-element code ABAQUS.[9] The analyses assume a rate-independent, J_2 (isotropic hardening) incremental plasticity theory, as indicated by the ABAQUS theoretical manual. From symmetry considerations, only half of the specimens are modeled. A typical finite-element mesh, which is shown in Fig. 3, is made of about 2300, 8-noded generalized plane-strain isoparametric elements. To simulate a very refined crack-tip region, the crack tip was surrounded by 40 "rings" of elements up to a distance of half the crack length. This highly refined crack-tip zone is needed to determine not only the J-integral, but also the associated crack-tip stress fields. Values of the J-integral as a function of the applied

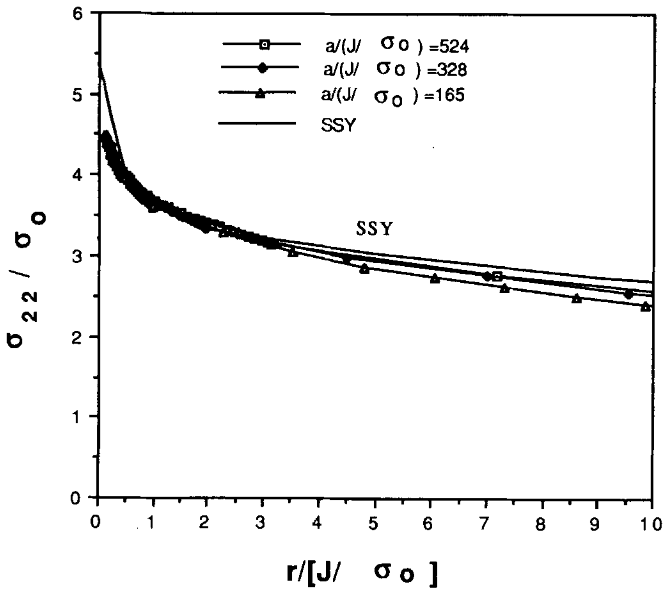

Fig. 4 Normal component of the crack-tip stress field obtained at various applied loads ($a/W = 0.5$).

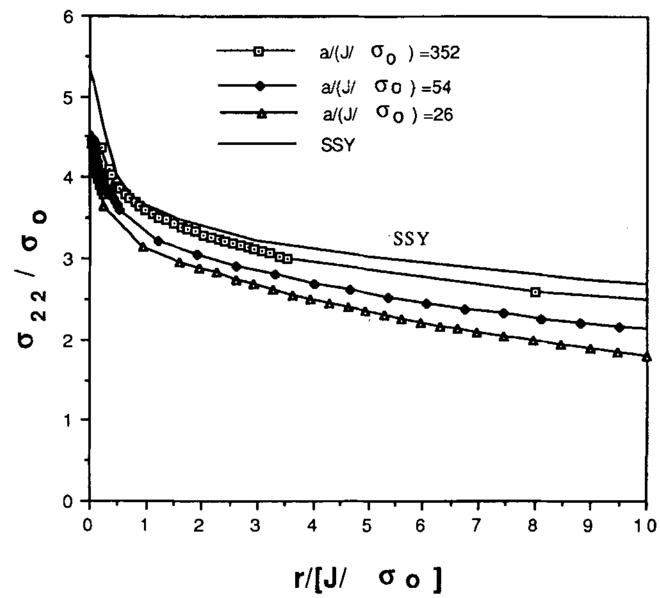

Fig. 5 Normal component of the crack-tip stress field obtained at various applied loads ($a/W = 0.3$).

loads were evaluated along 15 consecutive contours surrounding the crack tip. All contours, except the first, yielded the same J-integral value, with only about 0.1 to 0.5% discrepancy. Other convergence requirements were specified by means of limiting the residual forces (nonbalanced nodal forces) to a minimum and comparing the stress-intensity factors obtained from the elastic portion of the stress-strain data to those from a standard crack handbook.[10]

4. *J-Q* Methodology

The two-parameter approach (J-Q) presented by O'Dowd and Shih[7] provides a convenient way to measure the crack-tip fields under fully plastic conditions. Alternatively, the two-parameter approach (J-T) provided by Betegon and Hancock,[8] Parks,[11] and Parks and Wang[12] appears to be equivalent with some limitations. Details of the difference between the two approaches were discussed by O'Dowd and Shih.[13]

A key outcome of the J-Q methodology is the ability to define a collective crack-tip family of fields characterized by the Q parameter. The Q-parameter (or Q-stress) is defined as the difference between the normal (opening mode) stress component obtained from a plane-strain finite-element analysis and the corresponding reference field obtained from small-scale yielding conditions (SSY) at some distance r ahead of the crack tip, the difference being normalized by the yield stress in uniaxial tension, σ_0.

$$Q = \frac{\sigma_{yy} - \sigma_{ssy}}{\sigma_0} \text{ at } r/(J/\sigma_0) = 2 \qquad [3]$$

The reference field can be obtained easily from a modified boundary layer formulation (MBL) (see Bilby et al.)[14] Similarly, within the definition of the Q-stress, the HRR fields may be substituted for the SSY reference field. However, it can be shown that a more rigorous description of the J-Q annulus is obtained when using the SSY reference fields. In the present finite-element analysis, small-strain theory is used to compute the normal stress fields ahead of the crack tip. Ideally, only a finite strain theory can accurately predict the stress field in the presence of a blunted crack tip. However, because the present calculation of Q-stress is evaluated at a normalized distance parameter of $r/(J/\sigma_0) = 2$, which lies outside the blunting zone, the small and large strain theory are invariably equivalent.

5. Crack-Tip Stress Fields

The normal components of the stress fields ahead of the crack tip are shown in Fig. 4 to 7, along with the reference field obtained from the MBL method. The stresses are normalized by the yield stress σ_0, whereas distances ahead of the crack tip, r, are normalized by (J/σ_0). Four crack aspect ratios are considered: $a/W = 0.3$, 0.5 (deep cracks), and 0.05, 0.1 (shallow cracks). In the case of deep cracks, the stresses are comparable to the reference field, even at loads that cause large-scale yielding. This observation can be seen from the evolution of the plastic zones, which remain confined to the ligament ahead of the crack tip (Fig. 8). On the other hand, for the case of the shallow cracks, the stresses were considerably less than the reference field over the entire range of loading. The corresponding equivalent plastic strain contours shown in Fig. 9 indicate that plastic deformation might have extended back to the crack-free surfaces even before the ligament became fully plastic. Similar observations were made by Al-Ani and Hancock[15] and Betegon and Hancock.[8]

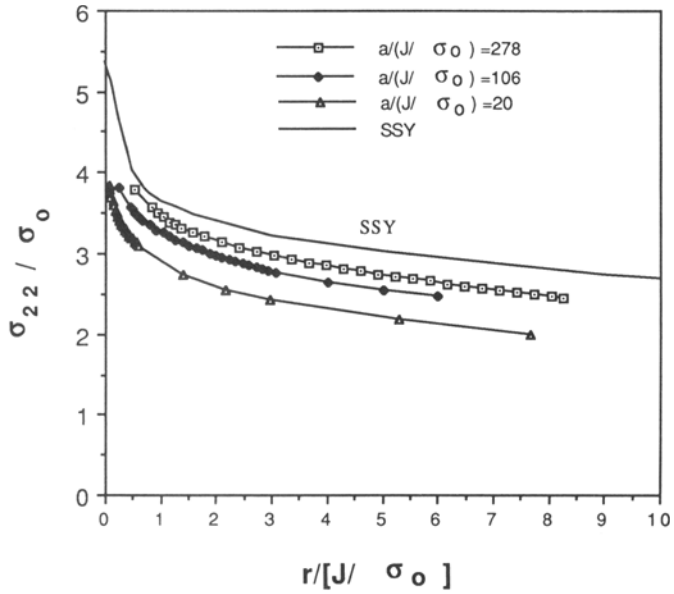

Fig. 6 Normal component of the crack-tip stress field obtained at various applied loads ($a/W = 0.1$).

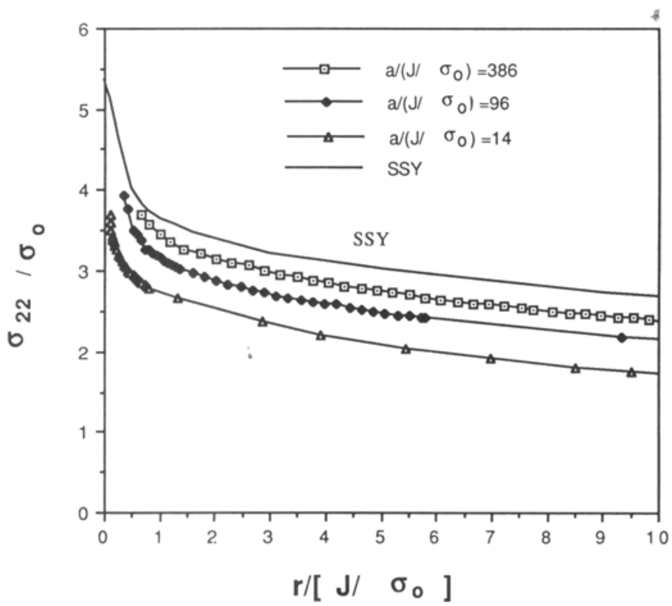

Fig. 7 Normal component of the crack-tip stress field obtained at various applied loads ($a/W = 0.05$).

6. *J-Q* Results

Following the methodology dictated by the (J,Q) approach, Fig. 10 illustrates four consecutive J-Q trajectories that correspond to each of the a/W ratios. From the results of deep cracks ($a/W = 0.3, 0.5$) it is clearly shown that the applied (J,Q) toughness locus depends weakly on the Q-stress over the entire range of loading. This is reflected by the crack-tip constraints, which are maintained even beyond the SSY conditions. In other words, a high crack-tip stress triaxiality is present throughout the loading process. On the other hand, toughness loci pertaining to the case of shallow cracks ($a/W = 0.05, 0.1$) revealed a

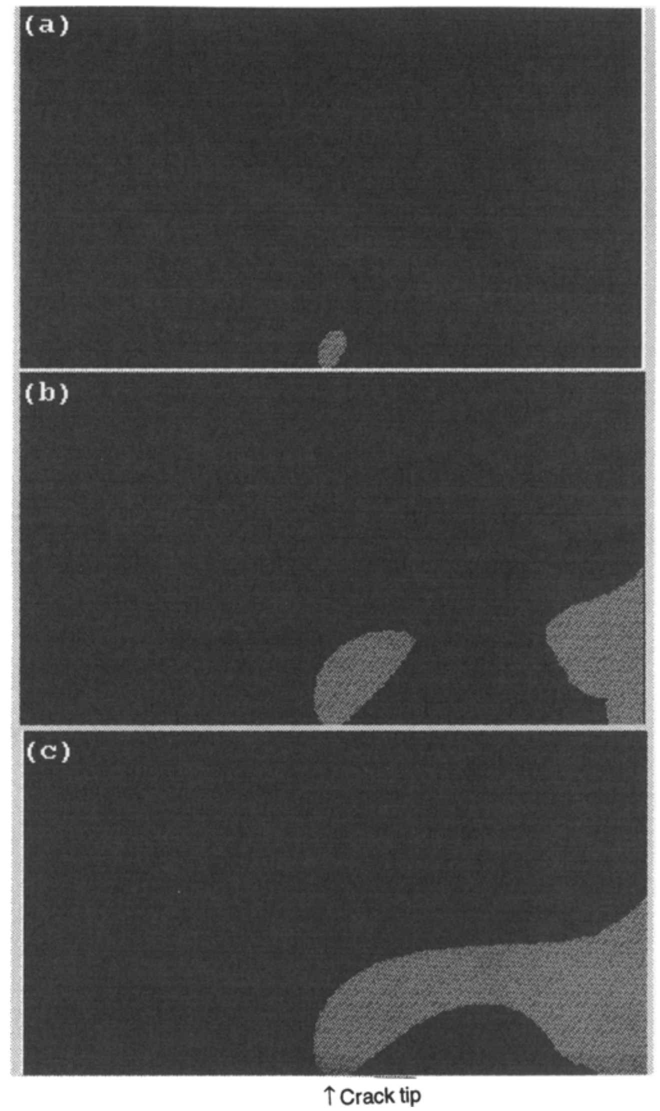

Fig. 8 Equivalent plastic contours at various applied loads ($a/W = 0.5$).

significant dependence on Q-stress. As previously mentioned, a negative Q-value indicates low crack-tip stress triaxiality and, therefore, loss of constraints, because a positive Q-value indicates high crack-tip constraint. Q equal to 0 corresponds to the special case where the crack-tip fields are equivalent to the SSY distributions. Although the toughness loci shown in Fig. 10 are constructed based on the calculation of the Q-stress at a single location, $r/(J/\sigma_0) = 2$, similar results were obtained when the Q-stress was evaluated at other locations, e.g., $r/(J/\sigma_0) = 4$ or 5. This indicates that perhaps restricting the calculation of the Q-stress at a single location is unnecessary as long as the condition $r/(J/\sigma_0) \geq 2$ is observed.

7. Discussion

The current data obtained from the (J,Q) methodology suggest the existence of a family of self-similar stress fields char-

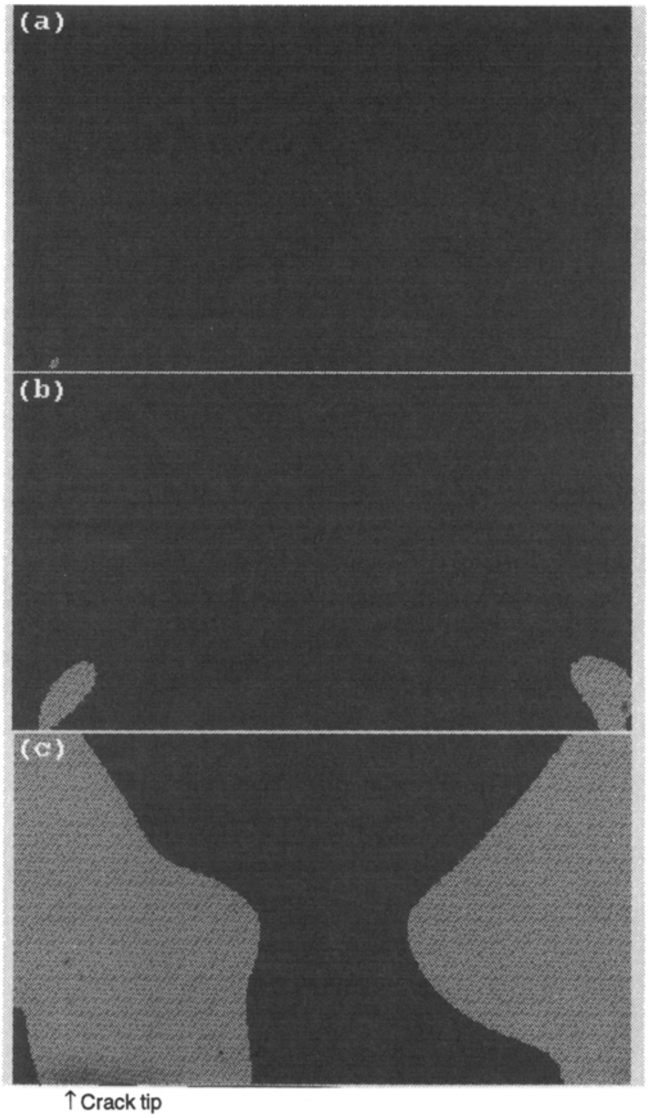

Fig. 9 Equivalent plastic contours at various applied loads ($a/W = 0.05$).

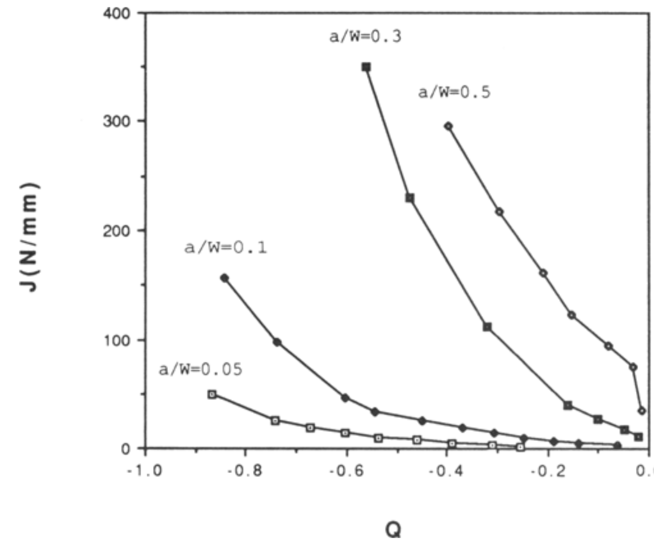

Fig. 10 Fracture toughness data measured with respect to the J-Q trajectories for each a/W ratio.

Table 1 Corresponding Q-stress values for various a/W ratios at the same applied load ($J/\sigma_0 = 0.125$)

a/W	Q
0.05	–0.791
0.1	–0.607
0.3	–0.167
0.5	–0.009

acterized by J and Q. As it is introduced by O'Dowd and Shih,[13] J and Q have two distinct definitions: J provides a size scale over which large stresses and strains develop, whereas Q represents a measure of crack-tip stress fields relative to a SSY reference field. As an example, Table 1 shows the different magnitudes of Q-stress that correspond to all four crack depths. Under the same applied J, ($J/\sigma_0 = 0.125$), Q varies from –0.009 for $a/W = 0.5$ to –0.791 for $a/W = 0.05$. This substantial difference is attributed to a significant loss of constraint in the case of a shallow crack ($a/W = 0.05$). The implication is that high and low crack-tip stress triaxiality can be rigorously quantified by the Q-stress alone. Alternatively, J-dominance criterion, such as the one developed by Shih and German,[6] suggested that crack-tip fields must lie within 90% of the HRR value at a distance of $r/(J/\sigma_0) = 2$. It follows that, for a SENB specimen, a ratio of $a/W = 0.3$ represents a transition between shallow and deep cracked geometries. However, the evolution from small-scale yielding to full plasticity may strongly depend on parameters such as the hardening exponent. Therefore, a transition between the one-parameter (J-dominance) and two-parameter (loss of J-dominance) criteria must be carefully addressed and examined for a given material.

8. Conclusion

For SENB specimens, it was found that the use of single-parameter criterion such as the J-integral was adequate for $a/W \geq 0.3$. This was observed by small magnitude of Q and plastic deformation confined to the ligament. For shallow cracks ($a/W < 0.3$), Q-values were significantly higher, even in the SSY conditions, and plastic deformation evolved back to the crack-free surfaces before the uncracked ligament became fully plastic. In the latter case, a two-parameter criterion for characterizing crack-tip fields, plastic deformations, and geometrical constrains is necessary. From the course of the present study, the (J,Q) approach seems to be suitable both from the computational aspect as well as the physical interpretation of experimentally determined fracture toughness values. In this spirit, the (J,Q) methodology can be regarded as a general elastic-plastic fracture criterion for which $Q \approx 0$ is a special case where J-dominance prevails.

Acknowledgments

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